





A FORTRAN ALGORITHM FOR PLOTTING CONTOURS OVER A TRIANGULAR MESH

Turbine Engine Division Components Branch

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FOREWORD

This report describes work conducted within the Air Force Aero Propulsion Laboratory, Turbine Engine Division, Components Branch (TBC), Wright-Patterson Air Force Base, Ohio. The work was accomplished under Project 3066, "Gas Turbine Technology," Task 06, "Turbine Technology," Work Units 02, "Turbine Aeromechanical Analysis," and 19, "Low Aspect Ratio Turbine Technology," between July 1978 and December 1978.

This report was submitted by the author in May 1979.

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1. INTRODUCTION

The use of digital computers to solve partial differential equations in two or more dimensions, e.g., in fluid dynamics or structural mechanics, typically involves the generation of numerical values for the dependent variable (or variables) at descrete points on a surface, which may or may not be a plane. If more than a few points are involved, graphical presentation of the results is virtually mandatory for human comprehension. Contour plots of the dependent variable convey information very effectively and hence are popular. Two algorithms which can be used when the descrete solution points are arranged on a rectangular grid are described in Reference 1.

However, not all problems are amenable to description on a rectangular grid. Finite element methods frequently use triangular elements and some solution domains simply are not rectangular. One also occasionally encounters data which is scattered irregularly over the surface. The algorithm described herein works very well in such environments. Though it requires a great deal more memory than LEVEL 1 or LEVEL 2 and requires more effort to set up, it is not difficult to use and produces high quality plots with little expenditure of computer time.

2. THE ALGORITHM

2.1 Overview

The algorithm presented here as ISOVAR2 and subsidiary routines ISOVAR3, FOLLOW, DRAWBND, DRAWL, and LABEL is a FORTRAN recoding of the Algol procedure tricont 2 described in Reference 2. It retains all the capabilities and limitations of the original. Because of the differences between FORTRAN and Algol, the method of invoking the algorithm is greatly different and is described in detail below and with some examples.

2.2 Operation

ISOVAR2 requires the user to supply several arrays and simple integers which describe the arrangement of the grid and the data on it. Some must be preset by the user while others may be treated as working storage. We shall describe them in order:

Integer working array dimensioned (E, 3).

EP

Integer array dimensioned (E, 3) in which EP (R,S), (S = 1, 2, 3,) are the subscripts in PX and PT of the vertices of element R, proceeding clockwise around the element. See Figure 1.

E

The number of triangular elements in the grid. Integer.

PX, PY, PZ

Real arrays dimensioned (G) containing the data to be contoured, X, Y, and Z values respectively. They may be doubly dimensioned in the calling program, in which case G is the product of the two dimensions. Note: PX and PY must be scaled to represent actual inches of pen movement on the plot. See the examples.

G

The number of points in each of PX, PY, and PZ. Integer.

HTS

Real array containing the contour heights to be drawn.

K

The number of contours in HTS. Integer.

LL, LP, LR

Integer working arrays, dimensioned (N, 2).

UNUSED

Logical working array, dimensioned (N).

N

Maximum number of lines bounding the triangles. Lines common to two triangles are counted once. N is never more than 3*G, nor less than E+G-1. See Figure 3.

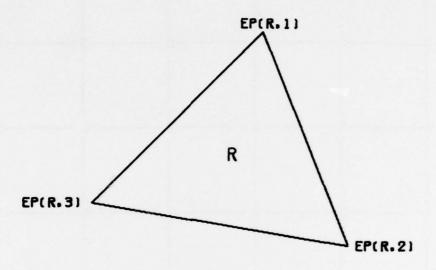


Figure 1
Definition of EP

ISOVAR2 will print the contents of the various arrays if the logical variable DBUG in labelled COMMON /BUGBUG/ is set .TRUE. This option should be used with discretion. Large meshes will yield very large quantities of printout.

2.3 A Simple Example

Consider an array of 30 Z values arranged on a 6 x 5 grid as shown in Figure 2. X values run from 0 to 5 and Y values from 0 to 4. Let us create a triangular mesh by running diagonals from upper left to lower right of each mesh rectangle as shown in Figure 3. Understand that this choice is completely arbitrary for this example and the other diagonal would have served just as well. In other applications, circumstances may favor a particular choice or combination of choices. In this example we could have taken some diagonals one way and some the other.

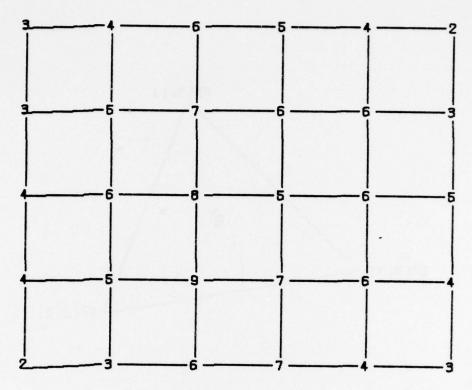


Figure 2 Sample Z Array

We can then create the PX, PY, and PZ arrays. From the description of the problem, G is 30. Thus:

DIMENSION PX(6, 5), PY(6, 5), PZ(6, 5)

DATA (PZ (I, 1), I = 1, 6) / 2.0, 3.0, 6.0, 7.0, 4.0, 3.0 /

...

DATA (PZ (I, 5), I = 1, 6) etc. DO 1 J = 1, 5

J1 - J - 1

DO 1 I = 1, 6

PX (I, J) = I - 1

PY (I, J) = J1

CONTINUE

Then the EP array is set up. From Figure 3, we know there are 40 triangles. Therefore E is 40. For our own reference (The information

is not passed explicitly to ISOVAR2) we need to adopt a numbering convention for the triangles and their vertices. Let us therefore start in the lower left corner of the mesh and proceed to the right, numbering the triangles in sequence, with odd numbers going to the lower triangles and even numbers to the uppers, as shown in Figure 3.

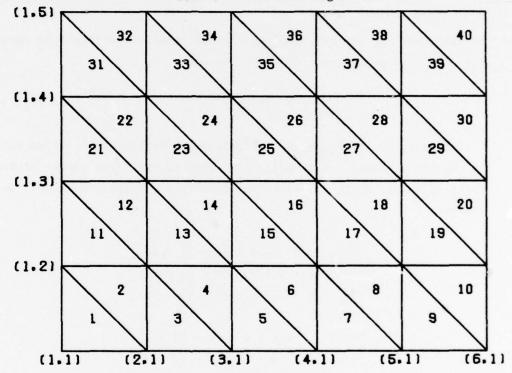


Figure 3
Triangular Mesh
Numbers in parentheses are
FORTRAN subscripts for PX,
PY, PZ
Numbers in triangles count the
mesh elements and correspond
to the first subscript R of
EP (R, S).

We also need a system for identifying the vertices of the triangles. Therefore, let us start at the lower left vertex of each lower triangle and at the bottom vertex of each upper triangle and count clockwise around each.

All that remains is to convert the double FORTRAN subscripts to single integers for EP. For this we use the same method FORTRAN uses: The relative location of Azj in an array dimensioned A(I, J) is given by

$$L = i - 1 + I * (j - 1)$$
 (1)

so that the location of the first element relative to itself is zero. Since we want equivalent subscript, we use

$$L 1 = i + I * (j - 1)$$
 (2)

The entries in EP are the equivalent subscripts of the three vertices of the Rth triangle. Thus EP(1, 1) points to the first corner of the first triangle, EP(1, 2) points to its second corner, and so on. In our example,

The choice of contours depends on the problem. We will use the sequence 2.5 to 8.5 in steps of 1.0, which makes K = 7.

Lastly, we must establish the size of the arrays LL, LP, LR, and UNUSED, i.e., the value of N. We know that N is not more than 90(=3*G) nor less than 69(=E+G-1). If we count the lines in Figure 3, we will find 69. For simple geometries like this, the lower limit will always suffice.

With this, we are ready to plot. The complete driver program is listed with ISOVAR2 in the Appendix and the completed plot is shown in Figure 4. Note that ISOVAR2 draws the grid boundaries; no special provision for those is necessary.

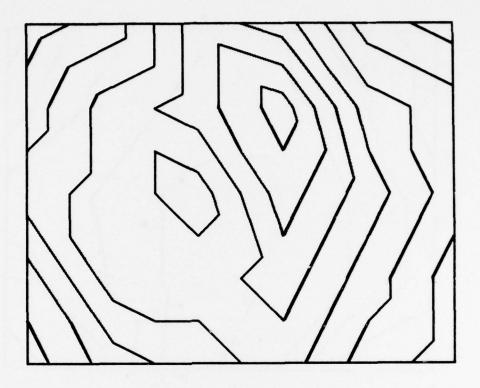


Figure 4
Sample Contour Plot

2.4 Multiple Data Sets on the Same Mesh

If one wishes to plot contours of more than one variable over the same mesh, he can bypass ISOVAR2 on the second and subsequent calls and enter the algorithm at ISOVAR3. The arguments are EL, EP, E, PX, PY, PZ, G, HTS, K, LL, LP, LR, UNUSED, and N. All have the same meanings as for ISOVAR2 except that PZ would be the alternate dependent variable set. The user must establish a new plot origin or the new plot will be superimposed on the old. Figure 5 shows a sample obtained by "relaxing" the first data set one cycle, i.e., for every point not on a boundary

ZP(I, J) = (PZ(I, J + 1) + PZ(I, J - 1) + PZ(I + 1, J) + PZ(I - 1, J)) / 4.0

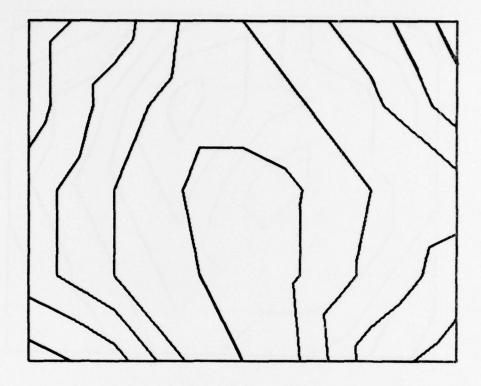


Figure 5
Contours of Second Data Set

2.5 An Example from Fluid Mechanics

The primary reason for developing ISOVAR2 from tricont 2 was to plot aerodynamic data at the exit of a turbine blade row. The general grid arrangement is similar to Figure 3, but with three differences. The upper and lower boundaries are circular arcs and the left and right boundaries are identical curves separated by the blade pitch angle. The grid is typically around 30×30 with density increasing toward each of the four walls. Figure 6 shows a typical plot of kinetic energy distribution, the highest levels being near the center.

Figure 6 required only five seconds of CDC 6600 central processor time to generate, but the total array storage (including PX, PY, and PZ) was more than 31000 (decimal) words.

2.6 Potential Improvements

2.6.1 Memory Requirements

As seen in the last example, one of the obstacles to the

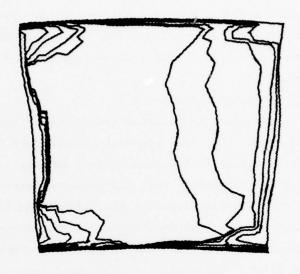


Figure 6
Kinetic Energy Contours at Stator Exit Plane

general application of ISOVAR2 is the large amount of memory it requires for the working arrays EL, EP, etc. Only the PX, PY, and PZ arrays are floating point and use an entire computer word for each element. The remaining arrays are integer or logical and therefore use only a fraction of a word per element. The EL and EP arrays for instance are dimensioned (E, 3). These could be packed by threes into a 60 bit CDC word. Allowing 18 bits each would permit a maximum value of 777777 octal or 262143 decimal, large enough for any application. The LL, LP, and LR arrays are dimensioned (N, 2). These could be packed two to a word or, by storing corresponding elements of LL, LP, and LR together, three to a word. Thus, half to two thirds of the integer array storage could be saved. The logical array UNUSED uses only one bit per word. These could be bunched 60 to a CDC word to save more than 98% of the required storage.

Packing and unpacking is a simple operation in CDC Fortran because it includes shift and mask instructions. IBM 360/370 Fortran allows a

length specification in type statements, e.g., INTEGER * 2 E, which specifies the number of bytes each variable is to occupy. Other computers may require assembly language subroutines for packing and unpacking. In any case, the modifications would require only a few hours' work, save much memory, and increase execution time by only a very small amount.

2.6.1 Labeling the Contours

Many times the plotted contours fall close together, double back, twist and turn, and generally become hard to distinguish. Figure 6 is a good example. For these cases, subroutine DRAWL can be modified to mark the beginning, end, and selected intermediate points of a contour with a distinctive symbol. The Calcomp routine SYMBOL has a repertoire of 15 suitable symbols which could be invoked by passing the value of the contour loop counter J from ISOVAR3 to DRAWL through a labelled COMMON block. Alternatively, one could cause DRAWL to print the X and Y coordinates at selected intervals on the contour. The latter method is used in the program which generated Figure 6. Annotating the contours directly is possible but would be difficult for the very reasons stated above.

3. CONCLUSIONS

ISOVAR2 has proven to be a useful and versatile algorithm which produces very satisfactory contour plots. It requires a lot of memory but very little execution time. The memory requirement can be reduced, as described in Section 2.6.1.

REFERENCES

- 1. J. S. Petty and K. D. Mach, Contouring and Hidden Line Algorithms for Vector Graphic Displays, AFAPL-TR-77-3, Air Force Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Ohio, January 1977.
- 2. B. R. Heap, Algorithms for the Production of Contour Maps Over an Irregular Triangular Mesh, NPL-NAC-10, National Physical Laboratory, Teddington, England, February 1972.
- 3. FORTRAN Extended Version 4 Reference Manual, Publication No. 60497800, Control Data Corporation, Sunnyvale, California, November 1975.
- 4. IBM System/360 and System/370 FORTRAN IV Language, Eleventh Edition, Publication No. GC28-6516-10, IBM Corporation, Palo Alto, California, May 1974.

APPENDIX PROGRAM LISTINGS

The complete FORTRAN code of ISOVAR2 and the driver program TWO which was used to generate Figures 4 and 5 are listed herein.

The listings contain occasional continuation lines marked with a dollar sign. These were created by the listing program to maintain the right-hand margin and do not appear in the actual code.

```
PROGRAM TWO (FLCT, OUTPUT=4028, TAPE6=OUTPUT)
      GIMFNSION H(7), LL(69,2), LF(69,2), LR(69,2), X(6,5),
     9 Y(6,5),
       2(6,5), 22(6,5)
      LOGICAL DRUG, U(69)
      INTEGER EF (40,3), FL (40,3), F, G, O
      COMMON /BUGPUG/ DBUG
C
      DATA H / 2.5, 3.5, 4.5, 5.5, 6.5, 7.5, 8.5 /
      DATA (Z(I,1),I=1,6) / 2.0, 3.0, 6.0, 7.0, 4.0, 3.0 /
      DATA (Z(I,2),I=1,6) / 4.0, 5.0, 9.0,
                                                7.0,
                                                      6.0,
                                                           4.0 /
      DATA (Z(I, 3), I=1,6) / 4.7, 6.0, 8.0, 5.0, 6.1,
                                                           5.0 /
      DATA (Z(I,4), I=1,6) / 3.0, 5.0, 7.0, 6.0, 6.7, 3.0 /
      DATA (Z(1,5),T=1,6) / 3.0, 4.0, 6.0, 5.0, 4.0, 2.0 /
C
       K(I, J)
                   = 1 + 6 * (J - 1)
C
        00 1 I
                     = 1, 6
         FI
                     = 1 - 1
           DO 1 J
                       = 1, 5
                       = FI
           X(I,J)
           Y(I,J)
                       = J - 1
    1
           CONTINUE
C
      DBUG
                   = .FALSE.
      E
                   = 40
      G
                   = 30
      N
                   = 60
      0
                   = 7
C
C
           LOAD EF WITH X, Y, AND Z SUBSCRIPTS
C
         CO 5 I
                     = 1, 5
           C 5 00
                       = 1, 4
00000
           LOWER TRIANGLE HAS SUESCRIPT & IN EP: UPPER HAS
                 SUESCRIPT L.
                       = 2 * (I + 5 * (J - 1))
                                                             THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COFY PURMISHED TO BOO
                       = L - 1
           EP (M, 1)
                       = K(I, J)
           EP (M, 2)
                       = K(I,J + 1)
           EP (M, 3)
                       = K(I + 1, J)
                       = K(I + 1,J)
           EP(1,1)
           EP (L, 2)
                       = K(I,J + 1)
           FP(L,3)
                       = K(I + 1, J + 1)
           CONTINUE
C
      CALL PLOT (2.0, 4.0, -3)
```

```
CALL ISOVAR2 (EL, FP, E, X, Y, Z, G, H, Q, LL, LP,
      $ LR, U, N)
C
C
           GENERATE SECOND DATA SET
         00 3 1
                      = 1, 6
                      = Z(T,1)
         Z2(I,1)
         Z2(I,5)
                     = Z(I,5)
    3 . CONTINUE
         nn 4 J
                      = 2, 4
         Z2(1,J)
                    = Z(1,J)
         Z2(6,J)
                     = 7(6, J)
         CONTINUE
         00 5 I
                     = 2, 5
           \begin{array}{lll} 00.5 & J & = 2, 4 \\ 7.2(I,J) & = 0.25 * (Z(Y,J+1) + Z(I,J-1) + \end{array}
     $ Z(I + 1, J) +
             Z(T - 1,J))
     1
    5
         CONTINUE
C
       CALL PLOT (10.0, 0.0, -3)
       CALL ISOVERS (X, Y, Z2, G, F, D, LL, LP, LR, U, N)
       CALL PLOT (8.5, +4.0, -3)
CALL SYMBOL (0.0, 0.5, 0.105, "FINISHED", 90.0, 8)
       CALL PLOTE
       STOP
       END
```

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FROM CORY PURMISHED TO EDG

```
SURROUTINE ISCUARZ (EL, EF, E, FX, FY, PZ, G, HTS,
     $ K, LL, LP,
        LR, UNUSEC, N)
       CCMMON /BUGBUG/
                          DRUG
       COMMON
              /ISOLIF/ L, L6
C
C
          THIS ALGORITHM IS USED FOR CONTOURING OVER AN
C
                IRREGULAR,
THE PAPAMETERS FEQUIRED ARE:
          TRIANGULAR MESH.
            - THE NUMBER OF TRIANGULAR FLEMENTS.
                                                   INTEGEF.
        F
        EL
            - INTEGER WORKING AFRAY, DIMENSTONED (E, 3).
        EP
            - INTEGER ARRAY DIMENSIONED (E, 3), IN WHICH
     2
               FF (R, S),
               (S = 1, 2, 3), ARE THE SUECCEIPTS IN PX AND
               PY OF THE
     4
              CORNERS OF THE ELEMENT P. APPANGED CLOCKWISE
              AFOUND THE ELFMENT.
            - REAL AFFAYS CONTAINING THE DATA TO BE
     $
               CONTOURED.
        PY
        07
        G
            - THE NUMBER OF POINTS IN EACH OF PX, PY, PZ.
               INTEGER.
        HTS
           - REAL AFFAY OF CONTOUR HEIGHTS TO BE DRAWN.
        K
            - THE NUMBER OF CONTOURS IN HTS. INTEGER.
            - INTEGER WORKING ARRAYS, DIMENSIONED (N. 2).
        LL
        LP
        LE
        UNUSED - LOGICAL WORKING AFFAY, DIMENSIONED (N).
            - MAXIMUM NUMBER OF LINES BOUNDING THE
               TRIANGLES.
              NEVER MORE THAN 3 * G, NOF LESS THAN E + G -
00000
               1.
          EXAMPLE
C
          FOR A RECTANGULAR APPAY CONTAINING M FLEMENTS IN
00000000000
          X DIRECTION AND O ELEMENTS IN THE Y DIRECTION,
           = 2 * (M - 1) * (0 - 1)
          G = + + 0
          K = YOUR CHOICE
          N = F + 6
          A DETAILED DESCRIPTION OF THE ALGORITHM IS GIVEN
          SECTION 3 OF THE ORIGINAL PEPCET.
C
          ADAPTED FROM THE ALCOL PROCEDURE TRICONT 2, FOUND
               IN
```

```
(HPL-NAC-10) ALGORITHMS FOR THE PRODUCTION OF
000000000
                CONTOUR MAPS,
           B.R. HEAF (NATL. PHYS. LAB., TENDINGTON, ENGLANC)
           MARCH, 1972. PAGE 21.
           INITIALLY THE EUUNDARY LINE COUNT, LB, AND THE
                FLEMENTS
           OF THE APPAY EL APE SET TO ZERO AND THE TOTAL
                LINE COUNT,
           L, IS SET TO ITS MAXIMUM POSSIBLE VALUE.
       DIMENSION
                     HTS (K), LL (N,2), LF (N,2), LP (N,2),
     $ PX (G), FY
          (G), PZ (G)
       INTEGER
                     E, EL (E,3), EF (E,3), G, FA, FB, PC, F,
     $ 5, SA, SB
       LOGICAL
                   DEUG, UNUSED (N)
       LE
                   = 3 * E
                   = LE
       L
       LB
                   = 0
          00 1 R
                     = 1, F
            00 1 5
                       = 1, 3
            FL(F, S)
                       = 0
            CONTINUE
    1
C
00000
           FACH ELEMENT OF THE MESH IS EXAMINED IN TURN, ITS
                FOINT
           LABELS EXTRACTED, AND THE APPROFRIATE ENTRIES IN
                THE
           ARRAY EL MACE USING SUBROUTINE LABEL.
C
          00 2 R
                     = 1, F
                     = FP(F, 1)
          PA
          PR
                     = EF(F, 2)
          PC
                     = EF(R, 3)
          IF (EL (P,1) .EQ. 0) CALL LABEL (P,1,P3,PC,E,FP,EL)
            (EL (R,2) .EO. 0) CALL LABFL (F,2,PC,PA,E,EP,EL)
          IF
          IF (EL(R,3) .EO. 0) CALL LABEL (P,3,PA,PE,E,EP,EL)
    2
          CONTINUE
C
           THE LINE COUNT, L, IS AMENDED AND THE LINE LASELS ADJUSTED SO THAT THE BOUNDARY LINES HAVE LASELS
CCCC
                 1,2,3,
           ..., LE, AND THE INTERIOR LINES HAVE LABELS LE+1,
C
                LB+2,
C
           . . . , L .
C
        IF (L3 .E0. LE) 60 TO 4
       LC
                   = LE - LB
```

```
= 1 - 10
          00 3 P
           0.3 P = 1, E

0.03 S = 1, 3
           IF (EL(R,S),GT,LB) FL (F,S) = EL (R,S) - LC
    3
           CONTINUE
C
0000
          THE DIMENSIONS OF THE AFFAYS LP, LL, LP (L, 2)
                FEGUIFED
          FOR ISOVARS ARE NOW KNOWN.
           INITIALLY ALL ENTRIES IN THE ARRAYS LR AND LL
C
           APE SET TO ZERO.
    4 CONTINUE
       IF (L .LE. N) GO TO 5
PRINT *, "INCREASE N TO ", L
       STOP
    5 CONTINUE
         00 6 P
                    = 1, L
                    = 1, 2
           00 6 5
           LR(F, S) = LL(R, S) = 0
    6
           CONTINUE
C
CC
          THE ARRAYS LF, LR, AND LL ARE SET UP BY SCANNING
                THROUGH
C
         ALL ELEMENTS OF THE MESH AND EXTRACTING THE
C
                AFFROFFIATE
C
          ENTRIFS OF ARFAYS EF AND EL.
C
         DO 7 P
                    = 1, F
            00 7 5
                      = 1, 3
            MA
                      = EL (R, S)
            SA
                      = 5 + 1
            IF
               (S \cdot FG \cdot 3) SA = 1
            SB
                      = S - 1
            IF
                (S .EO. 1) SB = 3
            NA
            IF
                (LR(MA,1) \cdot EQ \cdot Q) NA = 1
           LR(MA, NA) = EL(P, SB)
            LL(MA, NA) = EL(P, SA).
            IF (NA .NF. 1) GO TO 7
           LP(MA, 1) = EP(R, SA)
            LP(MA, 2) = EP(R, SB)
           CONTINUE
       IF ( .NOT. DBUG) GD TO 12
       PRINT 100
         DO 8 I
                    = 1, 5
         PPINT 101, I, (EP(I, J), J=1, 3)
         CONTINUE
       PRINT 102, L, LB
       PRINT 103
```

```
00 9 R = 1, L
         PRINT 104, R, LF (R, 1), PX (LP (R, 1)), FY (LF
     $ (P, 1)), LF
          (R, 2), PX (LP(F, 21), PY (LF (F, 2))
     1
         CONTINUE
       PRINT 105
         00 10 F = 1, L
         PPINT 101, F, LL (R, 1), LL (R, 2), LR (R, 1), LR
     £ (R, 2)
         CONTINUE
   10
       PRINT 106
         00 11 P = 1, E
         PRINT 101, K, (EL (R, S), S=1, 3)
         CONTINUE
   12 CONTINUE
C
          CONTOURS AFE NOW DRAWN USING SUPPOUTINE ISOVAF3.
C
C
       CALL ISOVARS (FX, FY, PZ, G, HTS, K, LL, LF, LR,
     4 UNUSFO, NI
       RETURN
  100 FORMAT ( 1H1, 9X, 1HI, 3X, 7HFP(I,1), 3X, 7HFP(I,2)
     $ , 3X ,
         7HEP(1,3) )
  101 FORMAT ( 1H , 5110 )
              ( *9 L =*, I3, * L8 =*, I3 )
( *1 LINE LIST* / 7X, 4HLINE, 6X, 4HFRCM,
  102 FORMAT
      FORMAT
  103
     $ 9x, 1HX, 9X,
     1 1HY, 8X, 2HTO, 9X, 1HX, 9X, 1HY )
  104 FORMAT ( 1H , I10, 2(I10, 2F10.1) )
  105 FORMAT ( #1 LINE LINKAGES* / 7Y, 4HLINE, 3X,
     $ 7HLL (P,1), 3X,
     1 7HLL (R, 2), 3x, 7HLR (R, 1), 3x, 74LR (P, 2) )
  106 FOFMAT ( *1 EOUNDAPY LINES* / 3X, 8HTRIANGLE, 3X,
     $ 7HEL (P,1),
       3x, 7HEL(R,2), 3x, 7HEL(R,3)
C
       END
```

```
SUBROUTINE ISCUAFS (PX, FY, FZ, G, HTS, K, LL, LF,
     $ LR, UNUSFO,
        N)
     1
       COHMON
               /AUGBUG/ DAUG
               /CLUFS/ OPEN, FIRST, LAST, H
       COMMON
               /ISOLIM/ L, La
       COMMON
C
C
          THIS PROCECURE IS A SUBPROCEDURE OF ISOVAR2 BLT
C
               CAN BF
C
          USED SEPARATELY. IT IS ALSO FOR CONTOURING OVER
C
               FN
C
          IRREGULAR TRIANGULAR MESH. THE PAPAMETERS
C
               REQUIRED ARE
C
          LB, THE NUMBER OF BOUNDARY LINES IN THE MESH,
C
              THE TOTAL NUMBER OF LINES IN THE MESH,
C
               INTEGER ARPAYS
C
          LP, LR, LL, WHERF LP (M, N), (N=1,2), ARE THE
Ç
               LAFELS OF THE
C
          POINTS AT THE ENDS OF LINE M, AND THE ENTRIES IN
C
               LR (M, N)
C
          AND LL(M,N) SPECIFY THE LABELS OF THE LINES
C
               JOINING AT
C
          THESE FOINTS. ALSO MEEDED ARE THE ARRAYS FX, FY,
C
     T
               AND PZ
C
          AND THE INTEGER K, WHICH ARE AS FOR ISOVAR2.
C
     4
               FULL DETAILS
C
          ARE GIVEN IN SECTION 3 OF THE OPIGINAL REPORT.
C
C
          THE BOUNDARY OF THE MESH IS NOW DRAWN.
C
       DIMENSION
                   HTS (K), L! (N,2), LF (N,2), LR (N,2),
     4 PX (G), PY
         (G), PZ (G)
       INTEGER
                   G, FA, PB, TEMF
       LOGICAL
                   DRUG, UNUSED (N)
       CALL DRAWBND (FX, PY, G, LF, N)
C
C
          EACH LINE IS EXAMINED IN TUFN, THE HEIGHTS OF ITS
C
               ENC
C
          POINTS EXTRACTED, AND THE ENTRIES IN THE ARRAYS
C
               LP, LR,
     $
          AND LL INTERCHANGED IF NECESSARY SO THAT CONTCURS
C
               APE
     $
C
          TRACED WITH FIGH GROUND ON THE RIGHT.
                   = 1, L
         00 1 M
                   = LF(M, 1)
         PA
         PB
                   = LF(M, 2)
         ZA
                   = FZ(FA)
```

```
ZR
                    = FZ(PB)
         IF (ZA .GT. ZF) GO TO 1
         LP (M, 1) = F8
                   = FA
         LD (m, 2)
                    = LR(M, 1)
         TEMP
                   = LR(M, 2)
         LR (M, 1)
        . LR (M, 2)
                   = TEMP
         TEMP
                    = LL(M, 1)
         LL (M, 1)
                   = LL(M, 2)
         LL (14, 2)
                    = TEMP
         CONTINUE
       IF ( .NOT. DBUG) GO TO 3
PRINT 100
         DO 2 M
                   = 1, 1
         PRINT 101, F, LF (M, 1), LF (M, 2), LL (M, 1), LL
     $ (M, 2), LR
           (M, 1), LF (M, 2)
         CONTINUE
       PRINT 102
    3 CONTINUE
C
C
          EACH CONTOUR HEIGHT IS CEALT WITH IN TURN.
C
         DO 6 J
                 = 1, K
                    = HTS(J)
C
C
          THE APPRY UNUSED IS SET UP FOR THIS HEIGHT,
C
               UNUSET (M)
C
          BFING SET . TRUF. IF A CONTOUR LINE OF THIS HEIGHT
C
          CROSSES LINE M.
           00 4 M = 1, L
           UNUSED (M) = PZ(LP(M, 1)) . GE. H. AND. PZ(LP(M, 2))
     $ .LT . H
           CONTINUE
C
C
          THE ARPAY UNUSED IS SCANNED FOR THE START OF A
C
                CONTOUR
C
          AND THE CONTOUR IS TRACED USING SUBROUTINE
C
                FOLLOW.
C
           DO 5 M
                   = 1, L
C
          WE USE I INSTEAD OF M HERE BECAUSE FOLLOW RESETS
C
          THIS INTEGER AND WOULD MESS UP THE LOOP SCUNT.
C
           IF (UNUSED(M) .AND. LP (M, 1) .NE. 0) CALL FOLLOW
       (I,PX,PY,FZ,G
              , LL, LF, LR, UNUSED, M)
```

```
SUBPOUTINE FOLLOW (M, PX, PY, FZ, S, LL, LP, LR,
     $ UNUSED, NI
       CCMMON
               /BUGBUG/ DBUG
       COMMON /ISOLIM/ L, LE
       COMMON /CLUES/ OPEN, FIRST, LAST, H
C
C
          THIS SUPROUTINF IS USED TO FOLLOW A PARTICULAR
0000
               CONTOUR
          THROUGH THE MESH. THE FARAMETER M IS THE LABEL
               OF THE
          LINE ON WHICH THE CONTOUR BEGINS.
C
       DIMENSION
                   Lt (N,2), LP (N,2), LR (N,2), FX (G), FY
     $ (G), PZ (G)
       INTEGER
                   G, FA, PB, TEMP
                  DELG, FIRST, LIST, OFEN, UNUSED (N)
       LOGICAL
       MSTART
C
CCC
          THE PROCEDURE BEGINS BY SETTING THE INITIAL
               VALUES OF
          FIRST, LAST, AND OPEN.
C
       FIRST
                 = .TFUE.
       IPAJ
                 = . FALSE .
       OPEN
                 = M.LE.LE
       IF (DBUG) PRINT 100.H
C
C
          NEXT FOINT.
CC
          THE ECINT WHERE THE LINE CROSSES THE LINE M IS
          DETERMINED BY INVERSE LINEAR INTERPOLATION.
       PA
                 = LF(M, 1)
    1
       PB
                 = LF(M, 2)
       ZA
                 = FZ(FA)
       ZP
                  = P7 (FB)
       TA
                 = 0.0
       DN
                 = ZA - ZB
           (DN .ME. 0.0) TA = (H - ZB) / DN
       TF
                 = 1.0 - TA
       TB
       XF
                  = TA * PX(PA) + TB * PX(PB)
       YF = TA * PY(PA) + TR * PY(PB)
IF (DRUG) PRINT 101, XP, YP
C
          THE APPROPRIATE ENTRY IN THE ARRAY UNUSED IS SET
000
               TO
          .FALSE. UNIESS IT IS THE FIRST POINT OF A CLOSEC
C
                    IF IT IS THE FIRST POINT OF A CONTOLR,
C
C
          LABEL OF THE INITIAL LINF IS STORED. OTHERWISE.
C
                TESTS
     $
```

```
ARE CARRIEC CUT TO SEE IF T IS THE LAST POINT ON
C
               THE
C
          CONTOUR.
       UNUSED (M) = FIRST . AND . . NOT . OPEN
       IF (FIRST) GO TO 2
      LAST = (CFEN.AND.M.LE.LB)
     $ .OR. (.NOT.OPEN.AND.M.EG.MSTART)
       60 TO 3
    2 MSTART
C
          THE COORDINATES OF T AFF OUTPUT TO THE PROCEDURE
C
C
               DRAWL .
C
    3 CONTINUE
       CALL DRAWL (XF, YF)
C
C
          PROVICED THAT IT IS NOT THE LAST POINT, THE LABEL
C
               OF THE
C
          NEXT LINE THAT THE CONTOUR CROSSES IS DETERMINED
C
               AND THE
C
          ROUTINE GOES BACK TO FIND A NEW POINT T.
       FIRST
               = .FALSE.
       IF (LAST) RETURN
       TEMP
               = LF (M, 1)
C
          IN CASE WE END UP IN THE BOONIES
C
C
       IF (TEMP .EG. C) RETURN
       M
                = LL (M, 1)
       IF (UNUSED (TEMP)) H = TEMP
       GO TO 1
       FORMAT ( *0 X AND Y FOR CONTOUR*, G15.7)
  190
       FORMAT ( 5x, 2G15.7 )
  101
       END
```

```
SUBROUTINE DEAMBND (PX, FY, G, LF, N)
       CCMMON /ISOLJM/ L, LP
C
000000
          THIS PROCEDURE IS USED TO DRAW THE BOUNDARY OF
                THE MESH.
          EACH BOUNDARY LINE IS SCANNED IN TURN, THE
                CCORDINATES OF
          ITS END POINTS EXTRACTED, AND THE APPROPRIATE PEN
          MOVEMENTS MACE.
C
       DIMENSION LF (N,2), FX (G), FY (G) INTEGER G, FA
         DO 1 T
                    = 1, 18
         DA
                    = LF(I, 1)
         GALL PLOT (FX(PA), PY(PA), 3)
PA = LF(I, 2)
         CALL PLCT (FX(PA), PY(PA), 2)
         CONTINUE
   . 1
       RETURN
       END
```

```
SUBROUTINE LAGEL (R, S, FA, PB, E, EP, EL)
       COMMON /ISOLIH/ L, LE
C
C
          THIS PPOCECURF IS USED TO LABEL THE LINE JOINING
          POINTS PA AND PB AND OFFOSITE TO THE POINT EP (R,
C
C
C
          IN ELEMENT F, AND TO PUT THE APPROPRIATE LABELS
          IN THE AFRAY EL.
C
C
          ALL ELEMENTS J (J .GT. R) ARF SEARCHED FOR A FAIR
C
C
               OF
          POINT LABELS IDENTICAL WITH PA AND PB.
C
C
                   EL (F,3), EP (F,3)
       DIMENSION
                   E, FL, CP, PA, PB, QA, GB, QC, R, S, T
       INTEGER
       JR
                 = R + 1
         00 1 J
                   = J8, F
                   = EF(J, 1)
         OA
                   = EF(J, 2)
         03
                   = FF(J, 3)
         OC
                   = 3
         IF (PA .EO. QE .AND. PE .EQ. QA) GO TO 2
                   = 1
         IF (PA .EQ. GC .AMD. PR .EQ. QB) GO TO 2
                   = 2
         IF (FA .EC. CA .AND. PB .FQ. QC) GO TO 2
         CONTINUE
C
00
          NO PAIR OF LABELS HAS BEEN FOUND, SO THE LINE
               MUST BE
C
          A BOUNCARY LINE. THE BOUNDARY LINE COUNT, LB, IS
C
          UPPATER AND ASSIGNED TO ELIR, S).
C
       LP
                = 18 + 1
       EL(P, S) = LE
       RETURN
C
          INSIDE .
C
C
    2 CONTINUE
C
          A PATR OF LASELS HAS EEEN FOUND, SO THE LINE MUST
C
C
C
          AN INTERNAL LINE IN THE MESH. THE LINE COUNT, L,
C
               IC
C
          UPPATED AND ASSIGNED TO THE APPROPRIATE TWO
               ELEMENTS
C
          IN THE ARRAY FL.
C
                 = L + 1
       EL(R, S) = EL(J, T) = L
       RETURN
       END
```

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SUBROUTINE DEAWL (X, Y)
COMMON /CLUES/ OPEN, FIRST, LAST, H
LOGICAL FIRST, LAST, OPEN
IF (.NOT. FIRST) GO TO 1
CALL PLOT (X, Y, 3)
FETURN
1 CALL PLOT (X, Y, 2)
RETURN
END